INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE
M. Stat. (NB Stream) – Semester I
2016 – 2018
Linear Algebra and Linear Models
Mid-term Examination (Linear Algebra)

Total Marks: 40 **Duration:** 2 hours

1. Prove or disprove the following.

   (a) Let S and T be subspaces of V and $\mathbf{x}, \mathbf{y} \in V$. Then $\mathbf{x} + S \subseteq \mathbf{y} + T$ iff $S \subseteq T$ and $\mathbf{x} - \mathbf{y} \in T$. [4]

   (b) If A, B, C and D are singular matrices then $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is also singular. [4]

   (c) Let A be a square matrix over $\mathbb{C}$. Then $\rho(A^T A) = \rho(A)$. [4]

   (d) Let A be a square matrix. Then for every $\mathbf{b} \in \mathscr{C}(A)$, $A\mathbf{x} = \mathbf{b}$ has a solution belonging to $\mathscr{C}(A)$ iff $A = A^2$. [4]

   (e) Let A be an $m \times n$ matrix with $\rho(A) = m$ and B be an $n \times m$ matrix with $\rho(B) = m$. Then AB is nonsingular. [4]

   (f) Let C and D be g-inverses of A and B respectively. Then DC is a g-inverse of AB. [4]

2. Define Hermite canonical form (HCF). Let H be a square matrix of order n which is in HCF. Provide a g-inverse of H and show that it is a g-inverse of H. [2+2]

3. Let A and C be square matrices. Show that $D = \begin{bmatrix} A & B \\ \mathbf{0} & C \end{bmatrix}$ is idempotent iff A and C are idempotent, $ABC = \mathbf{0}$ and $(I - A)B(I - C) = \mathbf{0}$. [4]

4. Let $V = \mathbb{R}^3$ and $S = Span(\{(1, -1, 1)^T, (1, 1, 0)^T\})$. Provide a complement T of S in V. Compute the projections of $(1, 1, 1)^T$ and $(1, -1, 2)^T$ into S along T. [4]

5. Let A be a square matrix of order n. Show that there exists some integer $k, 1 \leq k \leq n - 1$, such that $\rho(A^k) = \rho(A^m)$ for all $m \in \mathbb{Z}$ with $m > k$. [4]